1967-05

Investigation of the wake behind a cylinder at coincidence of a natural frequency of vibration of the cylinder and the vortex shedding frequency

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Cambridge, Massachusetts; Massachusetts Institute of Technology

http://hdl.handle.net/10945/12505

INVESTIGATION OF THE WAKE BEHIND A CYLINDER AT COINCIDENCE OF A NATURAL FREQUENCY OF VIBRATION OF THE CYLINDER AND THE VORTEX SHEDDING FREQUENCY

by
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SUPERVISOR: PROF. P. LEEHEY MAY 1967

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by

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Submitted in Partial Fulfillment

of the Requirements

for the Master of Science Degree in

Naval Architecture and Marine Engineering

and the Professional Degree,

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at the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

May, 1967

Signature of Author: ______________________
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Thesis Supervisor

Accepted by: ______________________
Chairman, Departmental Committee
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by

Charles Laurence Ballou, Lieutenant
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Submitted to the Department of Naval Architecture and Marine Engineering on 19 May 1967 in partial fulfillment of the requirements for the degree of Naval Engineer and the degree of Master of Science in Naval Architecture and Marine Engineering.

ABSTRACT

Theory for aeolian tone intensity created by rigid circular cylinders has been found to agree quite well with experimental results. This investigation is to experimentally explore the wake of a cylinder shedding vortices at a frequency that is equal to one of the natural vibrational frequencies of the cylinder. Investigation was carried out in the following areas:

1. lateral spacial correlation of vortex shedding.

2. distance behind cylinder and value of RMS velocity at assumed position where vortices start to "roll up".

3. observation of the phase between velocity signals at two points along a cylinder.

ACKNOWLEDGMENTS

The author wishes to thank Professor Patrick Leehey for his keen interest and supervision throughout the work.

Financial support for the work was provided by the Naval Ship Research and Development Center under Contract NONR 3963(25), "Aeolian Tones Associated with Resonant Vibration".

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LIST OF SYMBOLS

ρ	=	free stream density
r,θ	=	polar coordinates (θ being the upstream direction)
S	=	$\frac{Nd}{U}$ = Strouhal number
d	=	cylinder diameter = .154 inches
N	=	vortex shedding frequency cps
U	=	free stream velocity
C_ℓ	=	lift coefficient
M	=	Mach number
ℓ	=	length of cylinder shedding vortices = 15 inches
ℓ_c	=	spanwise coherent length of vortices
I	=	acoustic intensity
γ	=	centroid of area under cross-correlation coefficient curve
f_n	=	natural frequency at nth. harmonic
R_e	=	Reynolds number = $\frac{Ud}{\nu}$
ν	=	kinematic viscosity
A,B	=	voltage output of Constant Temperature Anemometer
$R(z),R_{AB}(z)$	=	cross-correlation coefficient
x,y,z	=	rectangular coordinate system
x	=	downstream direction
y	=	perpendular to cylinder
z	=	along cylinder axis
u,v,w	=	velocity fluctuations in the x,y,z respectively

CHAPTER I

INTRODUCTION

The study of fluid flow past circular cylinders has been in progress since the latter part of the 19th century when Strouhal first showed that the vibrations causing the sound were transverse to the direction of the wind. The early interest in the phenomena was mainly concerned with the vortex shedding frequency as a function of Reynolds number.

Until recently, very little work has been done concerning the intensity of vortex shedding sounds known as aeolian tones. Phillips[1] and Etkin, Korbacher and Keefe[2] have derived theoretical expressions for the radiated sound intensity by starting with Lighthills[3] basic theory of aerodynamic sound as had already been extended to rigid bodies by Curle[4].

Phillips used the product of $\ell \, \ell_c$ to account for the fact that the force may not be coherent over the entire length of the cylinder. The use of the product $\ell \, \ell_c$ is valid if the body is long compared to the coherent length. His resulting equation for sound intensity is:

$$I = \frac{\rho}{16 r^2} S^2 \overline{C_\ell^2} M^3 U^3 \ell \, \ell_c \sin^2\theta \tag{1}$$

$(\bar{-})$ = indicated time averages

ρ = free stream density

r, θ = polar coordinates (θ being the upstream direction).

$S = \frac{Nd}{U_o}$ = Strouhal number

d = cylinder diameter

N = vortex shedding frequency

U = free stream velocity

C_ℓ = lift coefficient

M = Mach number

ℓ = length of cylinder shedding vortices

ℓ_c = spanwise coherent length of vortices

I = acoustic intensity

Etkin, Korbacher, and Keefe derived a more general equation that considers any length of coherent force.

In order to allow for longer lengths of coherent force, the centroid of the area under the correlation coefficient must be included. The effective length that is substituted for ℓ^2 is $\ell_c\ (\ell - \gamma)$ rather than $\ell\ \ell_c$.

γ = centroid of the area under the cross correlation coefficient curve. γ becomes small as the length of coherent force becomes small. Therefore, equation (1) is apporached for

small ℓ_c. The more general equation from Etkin, et al, shows:

$$I = \frac{\rho}{16 r^2} S^2 \overline{C_\ell^2} M^3 U^3 \ell (\ell - \gamma) \sin^2\theta \tag{2}$$

Experimental investigation of radiated sound intensity from circular cylinders when shedding vortex wakes has been made by Gerrard[5] and Keefe[6]. In general, these experimental results confirm the theory for rigid bodies especially in the lower Reynolds number region.

The flow past cylinders that are non-rigid bodies has not been investigated sufficiently to determine what happens when the vortex shedding frequency and a natural frequency of the vortex producing body coincide. This thesis is a report of experimental investigation of certain properties of the wake behind a cylinder shedding vortices at a natural vibrational frequency of the cylinder producing the vortices.

The theoretical equation mentioned earlier for sound intensity can be simplified for evaluation to:

$$I \propto \overline{C_\ell^2} (\ell - \gamma) \ell_c \tag{3}$$

which shows the three terms that may change for the situation of frequency coincidence C_ℓ, ℓ_c, and γ. Current work using the

same experimental set up is proceeding to obtain experimental values for sound intensity, lift coefficient, and any "locking together" of the two frequencies. Phillips[1] expresses the opinion that the correlation length may increase greatly when the vortex shedding frequency and the vibration frequency coincide due to a coupling of fluid motions at different axial positions caused by the cylinder motion. This supposition was explored experimentally and the results are presented here.

CHAPTER II

PROCEDURE

II.1. Lateral Spacial Correlation

The aeolian tone test rig, which is pictured in Appendix A, was used to hold a cylinder of .154 inches diameter and 15 inches in length under very high tension to produce the desired natural frequency. It was desirable to have one of the lower harmonic vibrational frequency coincide with the vortex shedding frequencies in order to eliminate as many nodes in the cylinder's vibration as possible. The cylinder was put under as high tension as the test rig was physically capable of holding. The table below gives the natural frequencies of vibration and the air speed and Reynolds number required to cause vortices to shed at these frequencies. Only the wake of the 2nd. and 3rd. harmonic was explored.

	N (cps)	U (fps)	$R_e = \frac{Ud}{\nu}$
2nd. harmonic	1030	62.5	4.57×10^3
3rd. harmonic	1520	83	6.07×10^3

The cross-correlation coefficient was computed in the following manner:

$$R_{AB}(z) = \frac{\sqrt{\overline{(A+B)^2}} - \sqrt{\overline{(A-B)^2}}}{4\sqrt{\overline{A^2}}\ \sqrt{\overline{B^2}}} = \frac{1}{4}\ \frac{4\ \overline{AB}}{\sqrt{\overline{A^2}}\ \sqrt{\overline{B^2}}} \tag{4}$$

$R_{AB}(z)$ = the cross-correlation coefficient as a function of spacial separation of the hot wire anemometers.

The cross-correlation coefficient is stated here to be only a function of the separation and not a function of where the separation is located along the cylinder. The investigation to determine if this coefficient and, in turn, the correlation length is independent of location along the cylinder was not carried out and is a subject for future investigation.

The values of R(z) were then plotted against z. See Figures [4], [5], [6], and [7]. A mechanical integrator was used to determine the area under the R(z) curve. This value is defined as one half the correlation length.

$$\text{i.e.}\quad \ell_c = \int_{-\infty}^{\infty} R_{AB}(z)\ dz \tag{5}$$

A plot of ℓ_c as a function of Reynolds number is shown on Figure [8].

In this range of Reynolds numbers, the Strouhal number is practically constant at .205. The cylinder consisted of a .091" diameter steel wire over which was placed a Teflon tube that brought the overall diameter to .154 inches.

This cylinder was placed in the open jet test section of the low turbulence wind tunnel in the M.I.T. Acoustics and Vibrations Laboratory. Figure [1A] shows the dimensions of the wind tunnel. It is more fully described in a report by Hanson[7].

For the correlation measurements one of the hot wire sensing elements was placed in a stationary position while the other sensing element was held on a traverse mechanism that could be moved in a direction parallel to the cylinder. The two hot wires were oriented parallel to the cylinder and positioned at the edge of the wake where the signal is periodic, but not turbulent. This position varied from speed to speed, but in general was about 1/2 inch back and 1/4 inch to one side of the center of the cylinder. See Figure [2]. The signals from these two Constant Temperature Anemometers (CTA) went to a correlation unit that produced a sum or difference output. The RMS output was recorded on a Graphic Level Recorder. Four outputs were recorded

$$\sqrt{\overline{(A+B)^2}}, \quad \sqrt{\overline{(A-B)^2}}, \quad \sqrt{\overline{A^2}} \quad \sqrt{\overline{B^2}}$$

See Figure [3]. A, B = voltage signals from the two CTA's.

To further check the correlation at coincidence of frequencies a dual beam oscilloscope was used to observe the phase relation of the signals from each hot wire.

To ensure that the vortex shedding frequency is coincident with a natural frequency, a sine wave signal from an oscillator, set on the desired natural frequency by an electronic counter, was displayed on the dual beam scope. The speed of the tunnel was then adjusted to cause the hot wire signal to match that frequency of the oscillator.

After setting the wind tunnel speed, the signals from both CTA's were displayed on the oscilloscope for observation of the phase between the two signals.

To further check the correctness of calculation of frequencies a dual beam oscilloscope was used to observe the phase relation of the signals from each probe.

To ensure that the vortex shedding frequency is coincident with a natural frequency, a sine wave signal from an oscillator, set to the desired natural frequency by an electronic counter, was displayed on the dual beam scope. The speed of the tunnel was then adjusted to cause the hot wire signal to match that frequency of the oscillator.

After setting the wind tunnel speed, the signals from both DISA's were displayed on the oscilloscope for observation of the phase between the two signals.

II.2. Position of Maximum RMS Velocity

To determine the position where it is assumed that the vortices start to "roll up," one hot wire anemometer was used with a two directional traverse mechanism. A search technique was used in the following manner. Starting very close behind the cylinder, the wake was traversed noting the position at which the maximum RMS velocity occurred. The value of RMS voltage and the DC voltage output of the anemometer was recorded at this location along with the position. The hot wire sensing device was then backed away from the wire a little further. The same traverse was made recording the same values at this point. This was repeated until the maximum value of RMS voltage was bracketed. Then, the exact position was located by searching within the bracketed space. A typical plot of how the maximum RMS voltage varies as a function of distance behind the wire is presented as Figure [9]. The maximum value is presumed to be the location of where the vortices start "rolling up".

A traverse behind the cylinder was performed at three speeds to have an idea of the properties of the wake. The results of RMS velocity and mean velocity are plotted and presented as Figure [10], [11], and [12].

CHAPTER III

DISCUSSION OF RESULTS

III.1. Lateral Spacial Correlation

As stated in the introduction, the equation for the intensity of aeolian tones is proportional to lift coefficient, correlation length, and centroid of the area under the cross-correlation coefficient curve as thus:

$$I \propto \overline{C_\ell^{\,2}} \; (\ell-\gamma)\ell_c \tag{3}$$

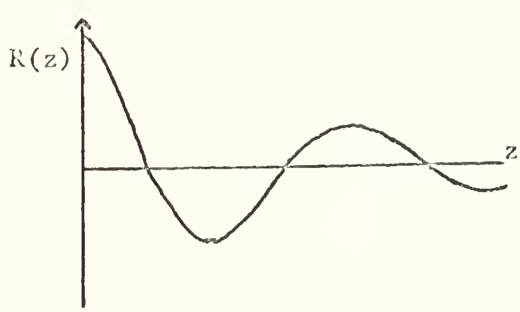
The correlation length ℓ_c is the effective length of a coherent vortex. As the Reynolds number increases, the two dimensional qualities of the vortex shedding start to disappear. In other words, there is a span-wise variation in the phase of the vortex shedding. The average coherent length of shed vortices is used in calculating the aeolian tone intensity. This average length is obtained by a procedure using two-point correlation of velocity along a cylinder as described in Procedure.

Correlation of velocity variations rather than force were made for two reasons. First, it was impractical to put pressure taps on the cylinder due to the small diameter. Secondly, because the cylinder has an induced motion the pressure tap readings would consist of a component due to the motion as well as the pressure caused by flow fluctuations.

el Baroudi[8] made similar cross-correlation measurements on a rigid cylinder. His data ranged in Reynolds number from about 10^4 to 4.5×10^4. He found that the correlation length increased as Reynolds number increased. His value of ℓ_c increasing from 3.2d at Reynolds number 10^4 to 6d at Reynolds number 4.5×10^4.

Figure [8], shows correlation length decreasing with increasing Reynolds number from about 8d at Reynolds number of 6.5×10^3 to 5d at Reynolds number of 11×10^3.

No apparent change in the trend of ℓ_c was noted at points of coincidence of the vortex shedding frequency and a natural vibrational frequency of the cylinder. It was thought that with the cylinder vibrating primarily in one mode that the coherent length would increase possibly approaching $\frac{\ell}{n}$ (n = mode number). Gerrard[9], in fact, measured the correlation coefficient at low Reynolds number and found it to be of the following shape:

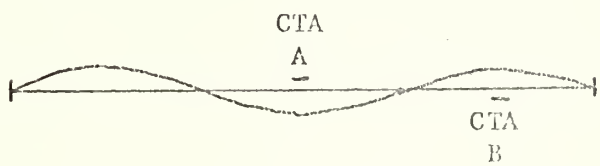


In the measurements obtained during this investigation, the value of the correlation coefficient approached zero monotonically at all velocities including those at which the vortex shedding frequencies coincided with a natural frequency of the cylinder.

More cross-correlation measurements should be made with particular emphasis on having the vortex shedding frequency exactly match the natural frequency and also more measurements at frequencies near the natural frequency.

When observing the signals from both hot wires simultaneously on a dual beam oscilloscope, with the hot wire probes adjacent to one another the signals remained inphase as would be expected. As the probes were moved apart parallel to the cylinder, the two signals did not remain inphase nor show a constant phase shift. Instead the two signals would move in and out of phase with no apparent pattern.

To check for phase reversal when crossing nodal points the hot wires were placed on each side of a nodal point at a point midway between the nodal points as thus for the 3rd. harmonic.



Again the signal moved continuously in and out of phase as previously described.

In all these observations, on the dual beam scope, there didn't appear to be any change of observed signal whether the vortex shedding frequency was in coincidence or not with a natural frequency.

III.2. Position of Maximum RMS Velocity

The position of the maximum RMS velocity is assumed to be the point where the vortex is "rolling up".

The maximum RMS voltage referred to is the unfiltered hot wire anemometer signal. The signal at the position of maximum RMS velocity contained random frequencies together with the vortex shedding frequency. A frequency spectrum was taken of the hot wire anemometer signal. The background noise was at least 15db lower at all times than the signal at the vortex shedding frequency. Therefore, the addition to the broad band signal was less than 0.5db or .01 volts.

As shown in Figure [9] the magnitude of the fluctuating velocity builds up linearly as the hot wire is moved downstream to a maximum then falls off showing the viscous diffusion of the vorticity.

Birkhoff[9] assumes that the wake acts as an oscillating flat plate attached to the cylinder providing alternating lift as the wake swings from side to side. He develops a satisfactory Strouhal number using this assumption. If it is assumed that the point of maximum RMS voltage is the point where the vortex starts "rolling up" and that Birkhoff's evaluation of the wake acting as an oscillator providing alternating lift, then the distance downstream to the point of maximum RMS voltage and also the

distance off to one side of centerline where the maximum RMS voltage occurs would affect the lift produced by the wake.

The distance downstream to the point of maximum RMS voltage is seen from Figure [13] to decrease linearly in the Reynolds number range considered. This follows the same trend as reported in Goldstein[10] from data taken by Schiller and Linke.

Again we see no deviation from this trend when the vortex shedding frequency and a nautral frequency coincide.

Figures [10, [11], and [12] show the mean velocity and RMS velocity of the wake at three different speeds with one of the situations having the vortex shedding frequency coincide with the 3rd. harmonic of the natural frequency. The results show that with Reynolds number increasing the maximum difference in mean velocity increases indicating an increase in vortex strength.

The maximum RMS velocity occurs at the point where the mean velocity is halfway between the maximum and minimum mean velocity. This is expected because this is the area of maximum shear flow. The slope of the mean velocity increases in the region of maximum shear flow with an increase in Reynolds number.

In conclusion, there did not appear to be any abnormalties in the properties of the wake when coincidence of frequencies occurred.

APPENDIX A

FIGURES

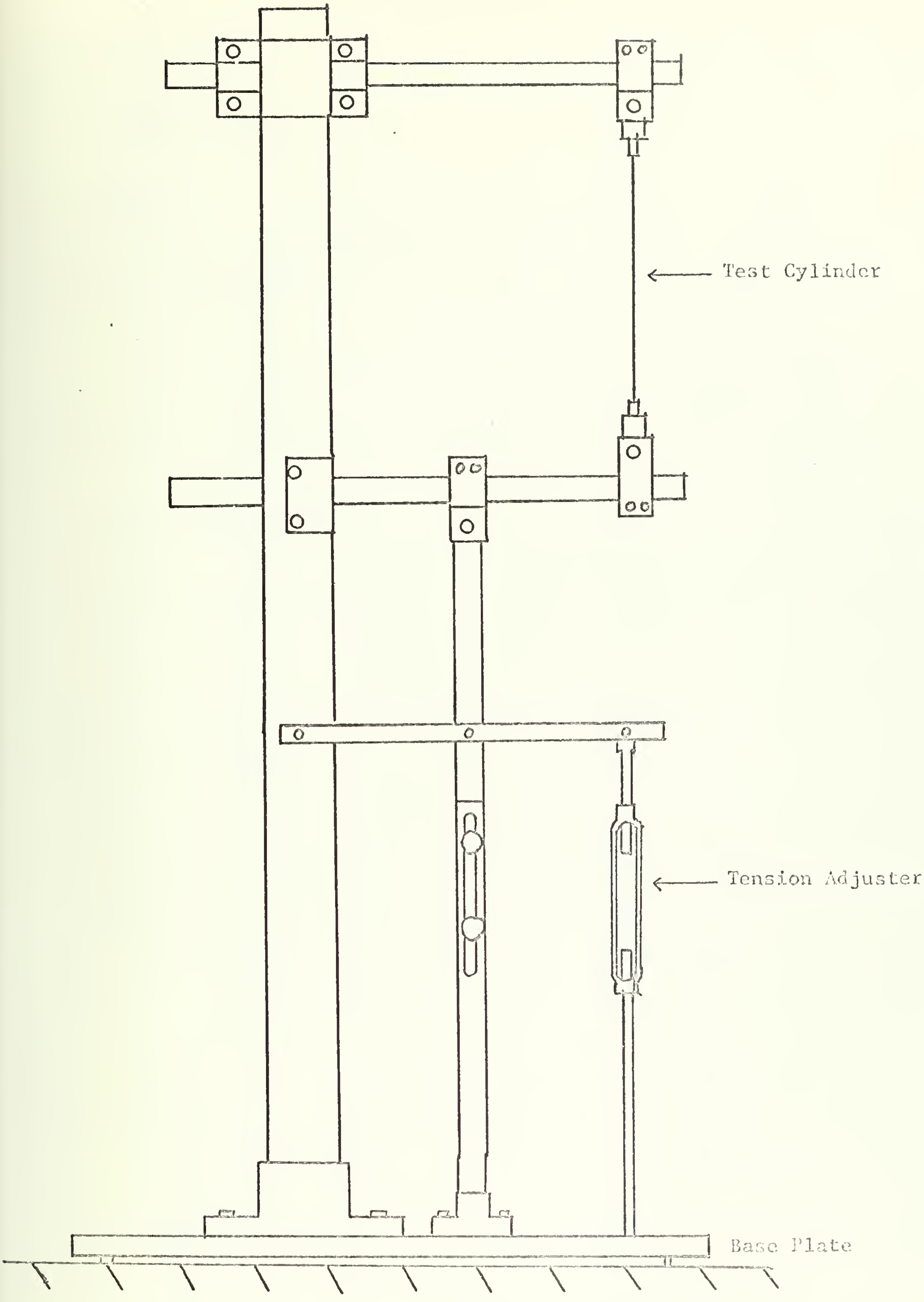
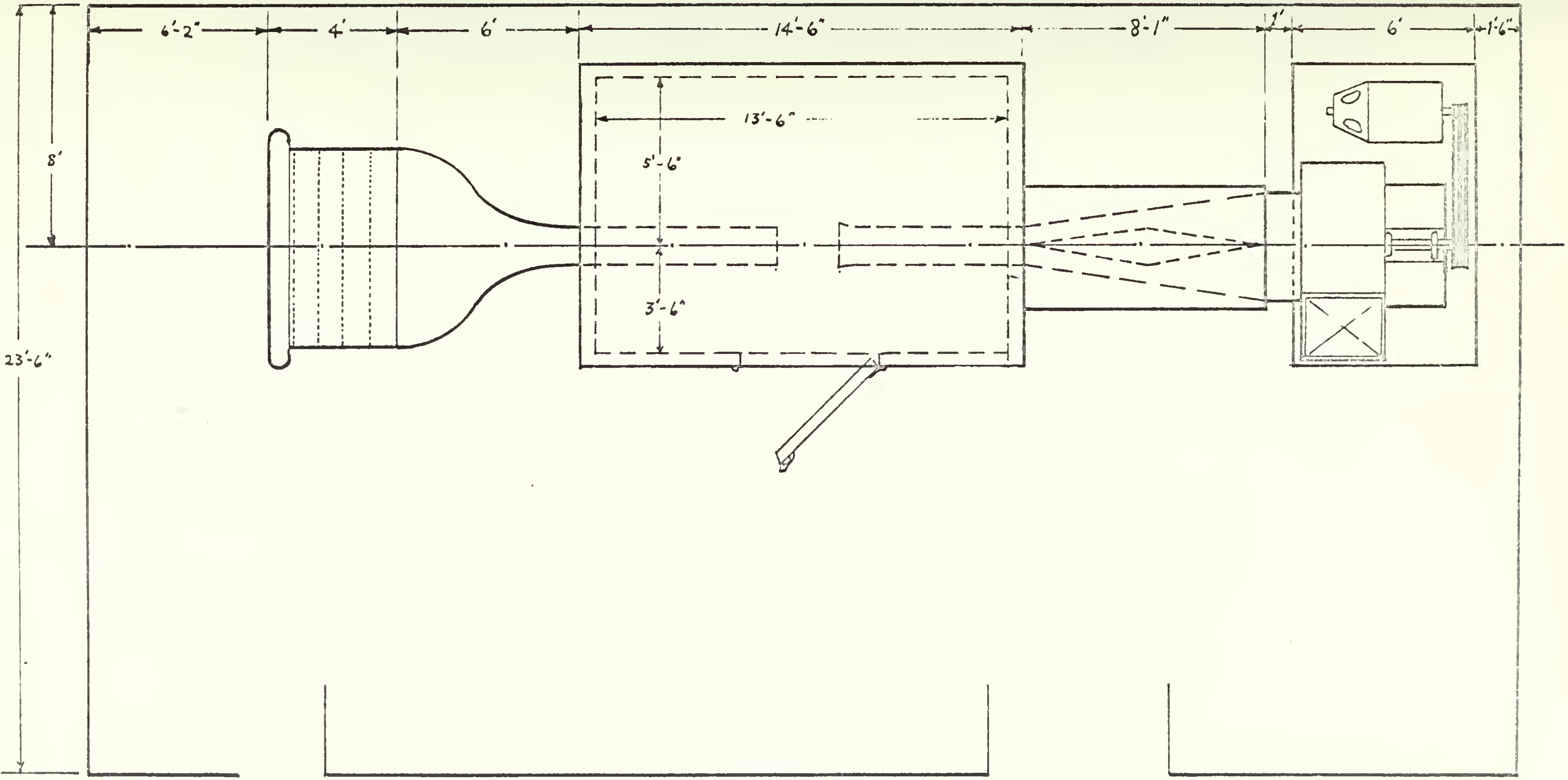
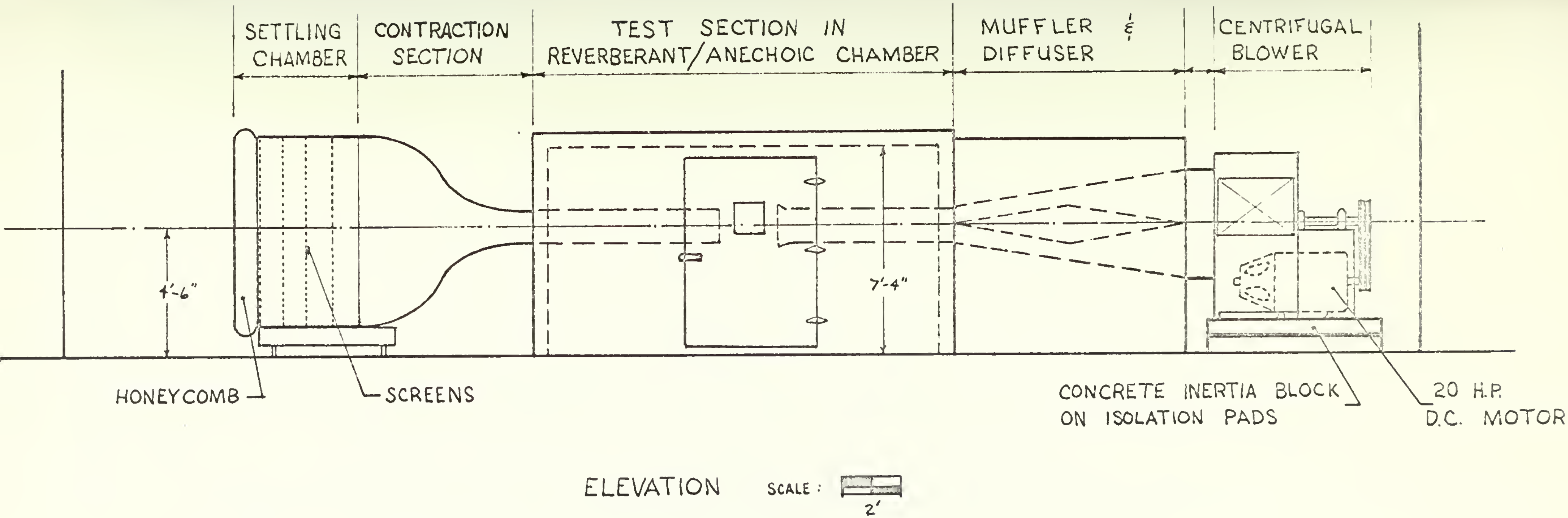


FIGURE 1

AEOLIAN TONE TEST RIG



PLAN VIEW SCALE : 2'



GENERAL SPECIFICATIONS :
CONTRACTION RATIO : 20 : 1
TEST SECTION : 15" x 15" SQUARE , OPEN OR CLOSED

FIGURE 1A

WIND TUNNEL FACILITY - ROOM 5-024
ACOUSTICS & VIBRATIONS LABORATORY
MASSACHUSETTS INSTITUTE OF TECHNOLOGY
CEH 8-23-66

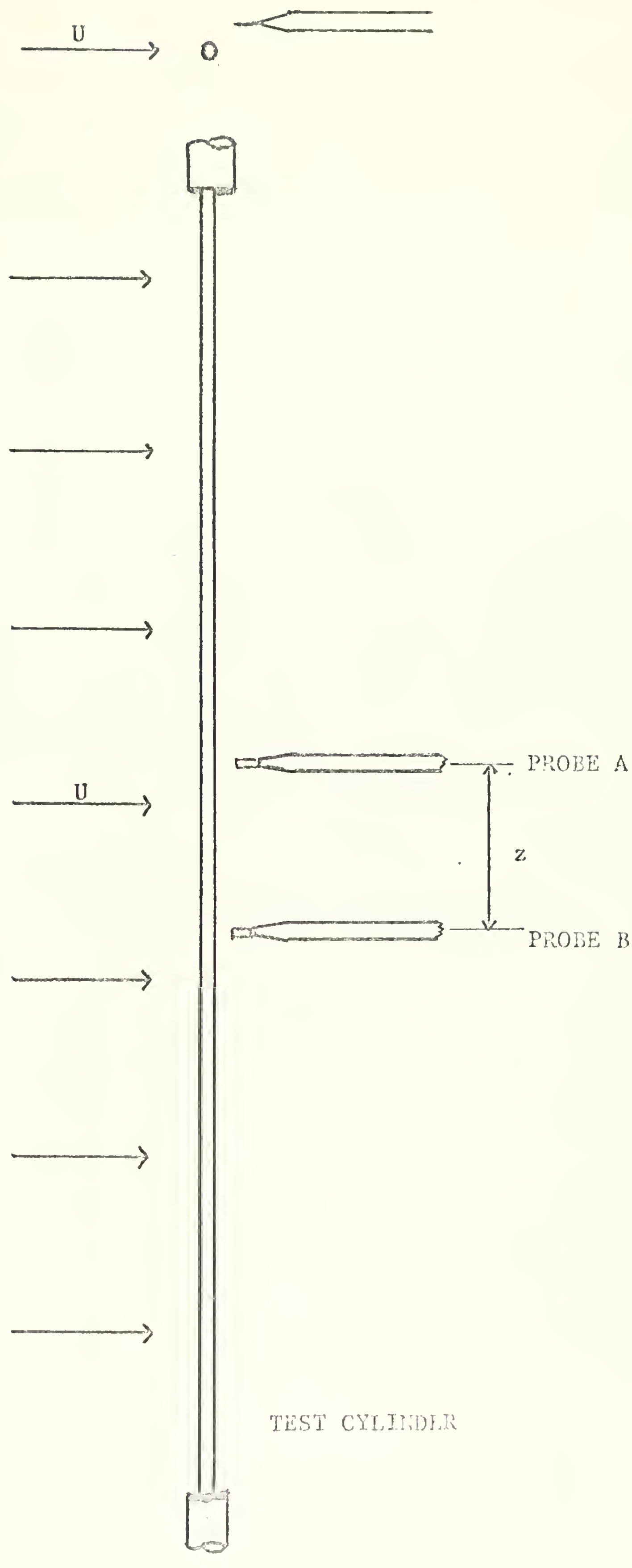
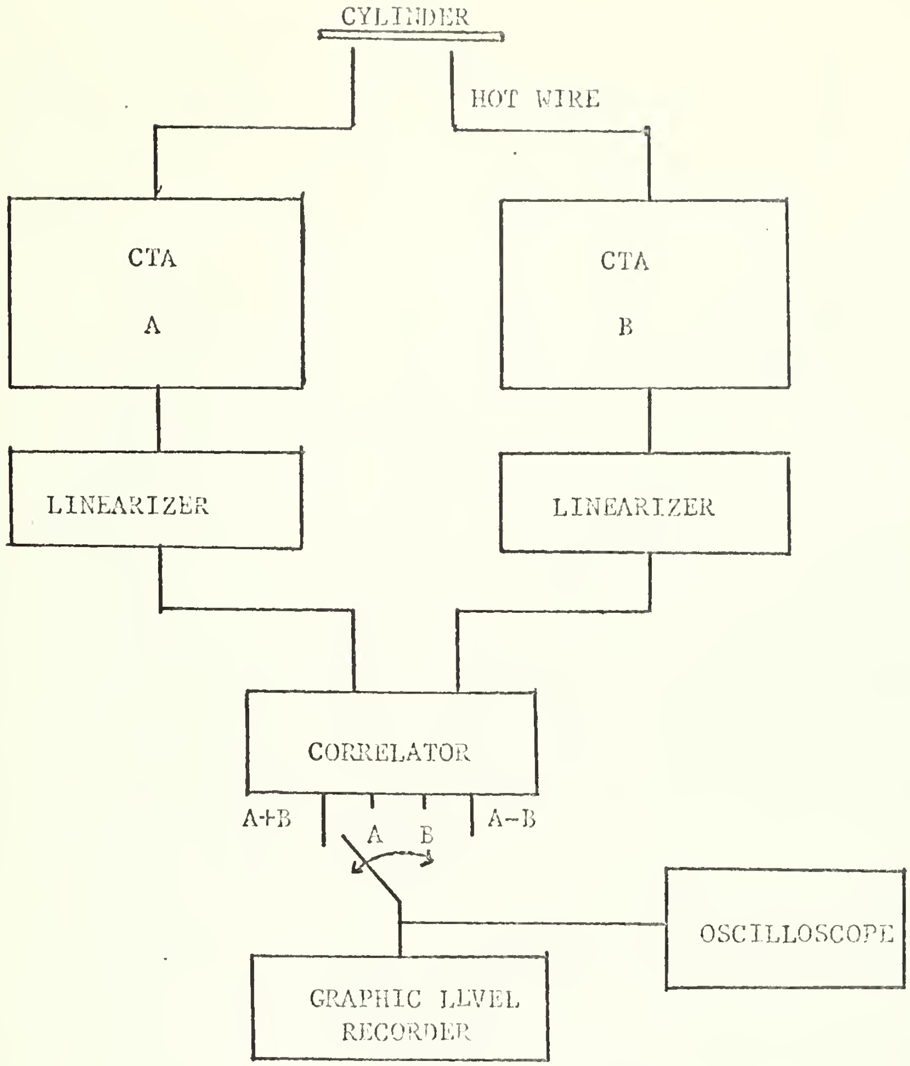
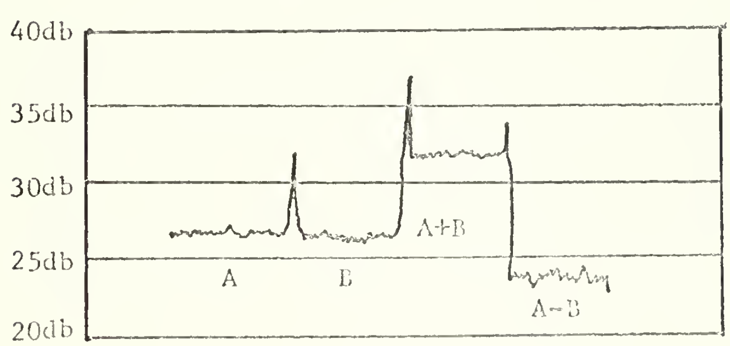


FIGURE 2

SKETCH OF THE LOCATION OF THE HOT WIRE PROBES IN RELATION TO THE TEST CYLINDER

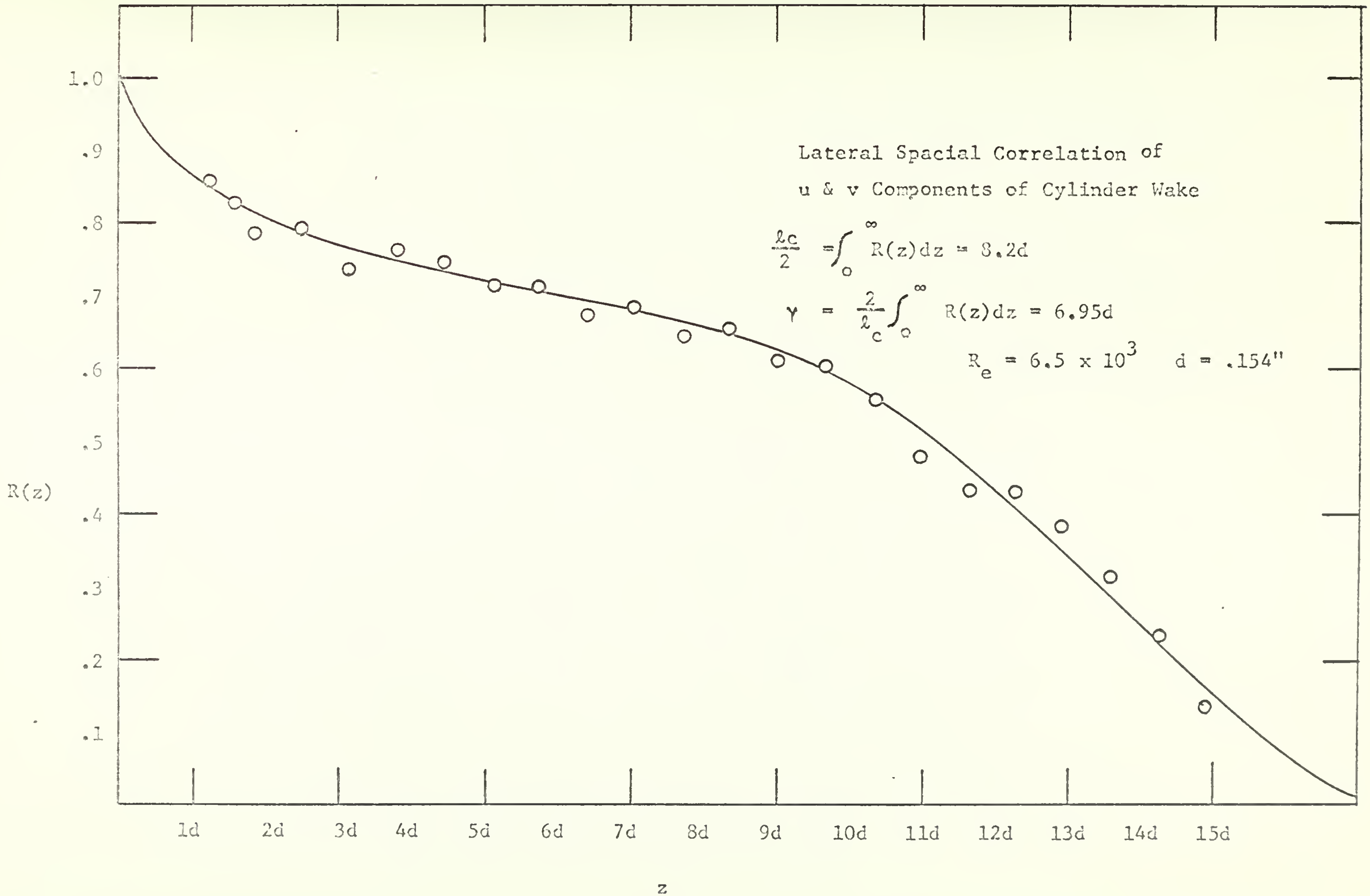


EXPERIMENTAL SETUP FOR CORRELATION CURVE DATA



SAMPLE OUTPUT FROM GRAPHIC LEVEL RECORDER

FIGURE 3



Lateral Spacial Correlation of
u & v Components of Cylinder Wake
$\frac{\ell c}{2} = \int_o^\infty R(z)dz = 8.2d$
$\gamma = \frac{2}{\ell_c}\int_o^\infty R(z)dz = 6.95d$
$R_e = 6.5 \times 10^3$ d = .154"
1.0
.9
.8
.7
.6
.5
R(z)
.4
.3
.2
.1
1d
2d
3d
4d
5d
6d
7d
8d
9d
10d
11d
12d
13d
14d
15d
z

FIGURE 4

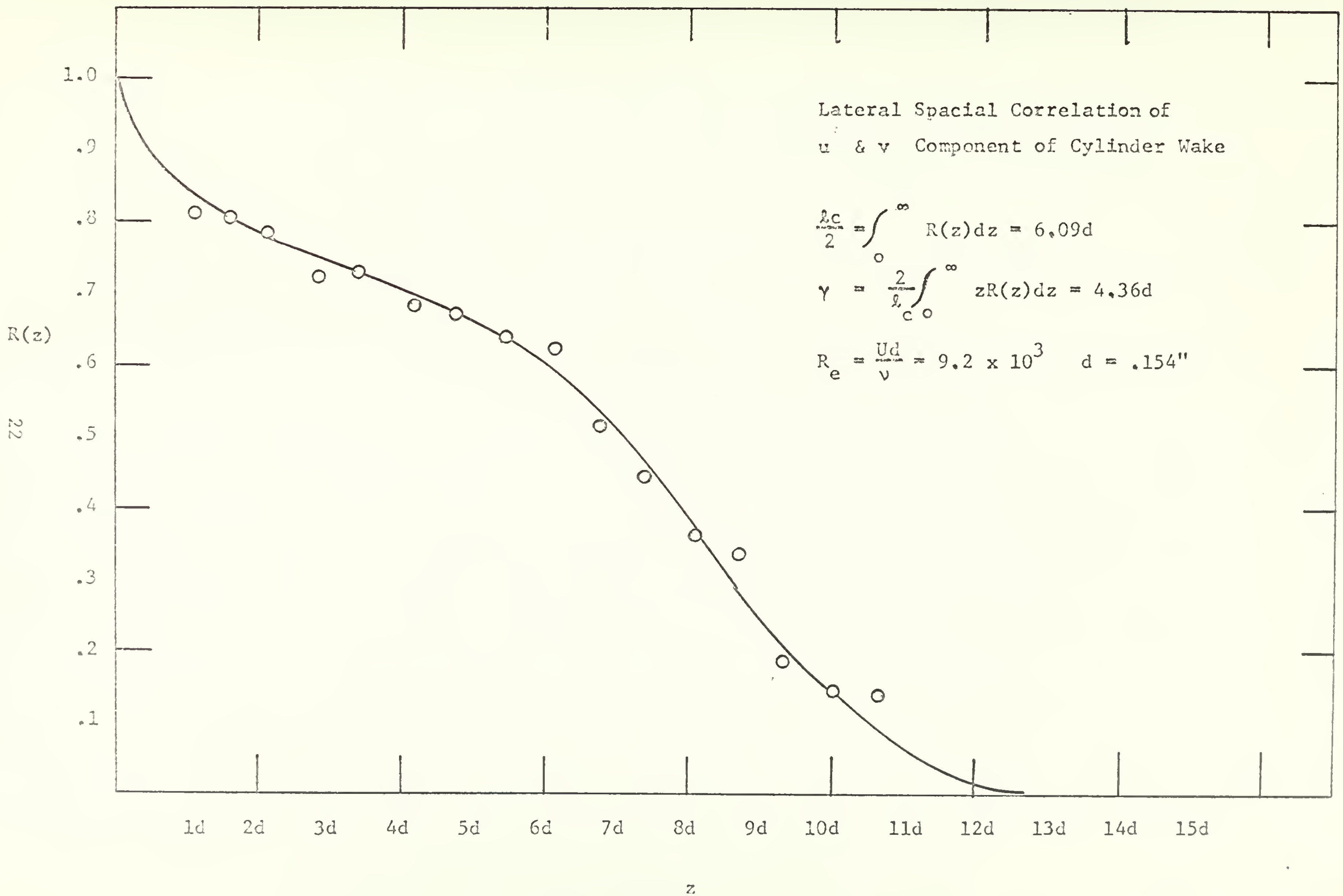

1.0
.9
.8
.7
.6
.5
.4
.3
.2
.1
R(z)
Lateral Spacial Correlation of
u & v Component of Cylinder Wake
$\frac{\ell_c}{2} = \int_0^\infty R(z)dz = 6.09d$
$\gamma = \frac{2}{\ell_c}\int_0^\infty zR(z)dz = 4.36d$
$R_e = \frac{Ud}{\nu} = 9.2 \times 10^3$ d = .154"
1d
2d
3d
4d
5d
6d
7d
8d
9d
10d
11d
12d
13d
14d
15d
z

FIGURE 5

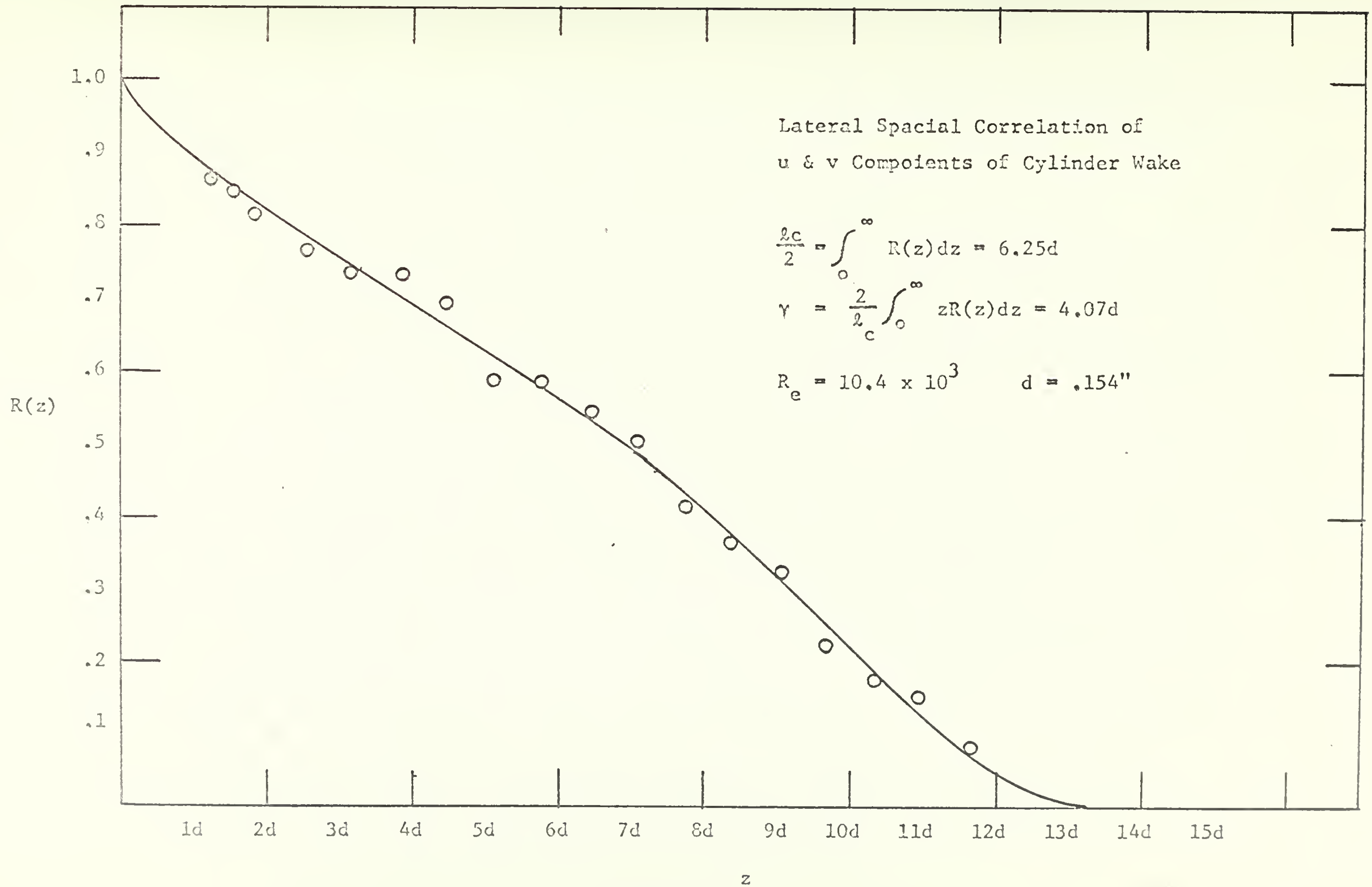

Lateral Spacial Correlation of
u & v Compoients of Cylinder Wake
$\frac{\ell_c}{2} = \int_0^\infty R(z)dz = 6.25d$
$\gamma = \frac{2}{\ell_c}\int_0^\infty zR(z)dz = 4.07d$
$R_e = 10.4 \times 10^3$ d = .154"
1.0
.9
.8
.7
.6
.5
.4
.3
.2
.1
R(z)
1d
2d
3d
4d
5d
6d
7d
8d
9d
10d
11d
12d
13d
14d
15d
z

FIGURE 6

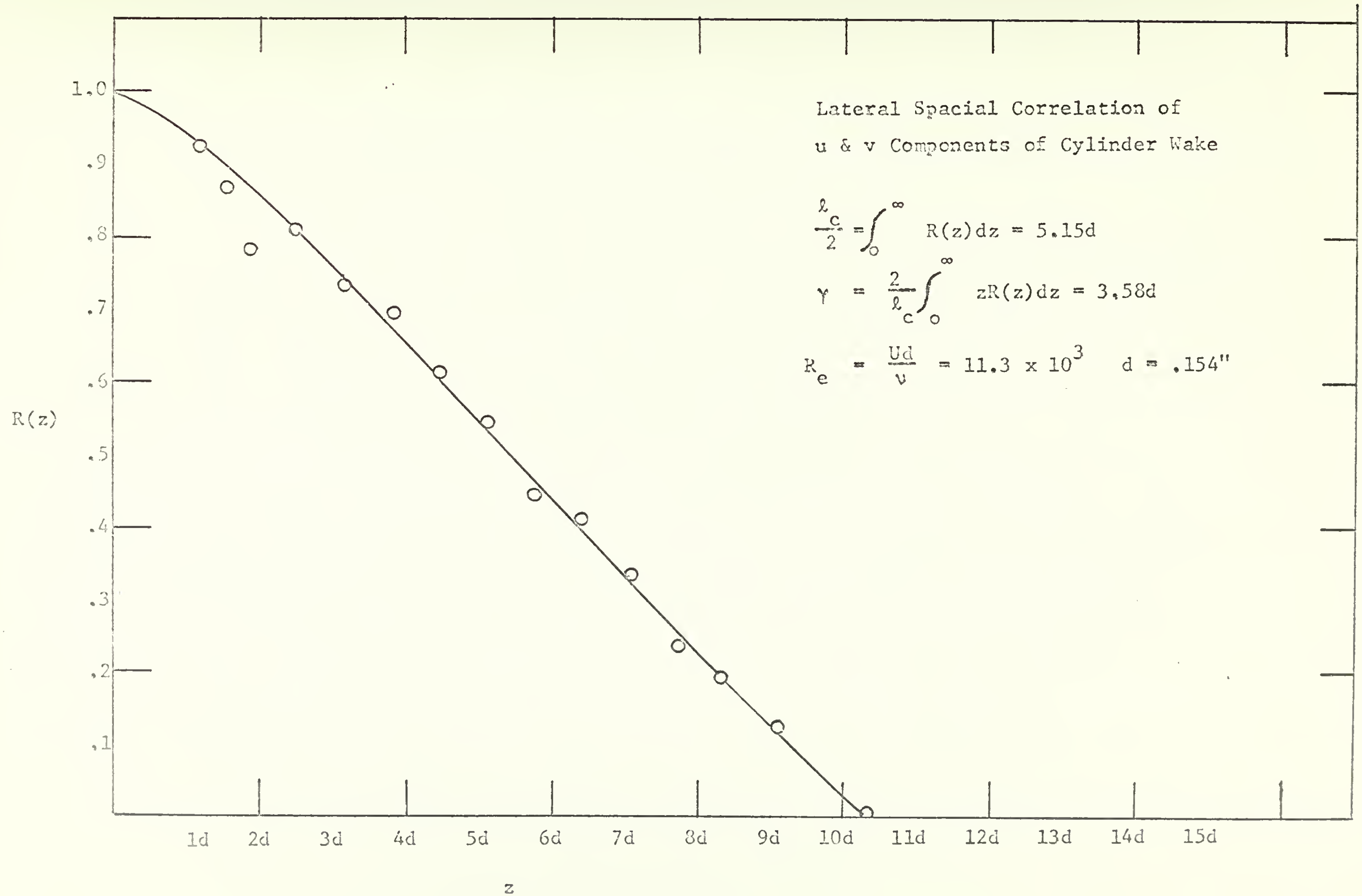

Lateral Spacial Correlation of
u & v Components of Cylinder Wake
$\frac{\ell_c}{2} = \int_0^\infty R(z)dz = 5.15d$
$\gamma = \frac{2}{\ell_c}\int_0^\infty zR(z)dz = 3.58d$
$R_e = \frac{Ud}{\nu} = 11.3 \times 10^3 \quad d = .154''$
R(z)
1.0
.9
.8
.7
.6
.5
.4
.3
.2
.1
1d
2d
3d
4d
5d
6d
7d
8d
9d
10d
11d
12d
13d
14d
15d
z

FIGURE 7

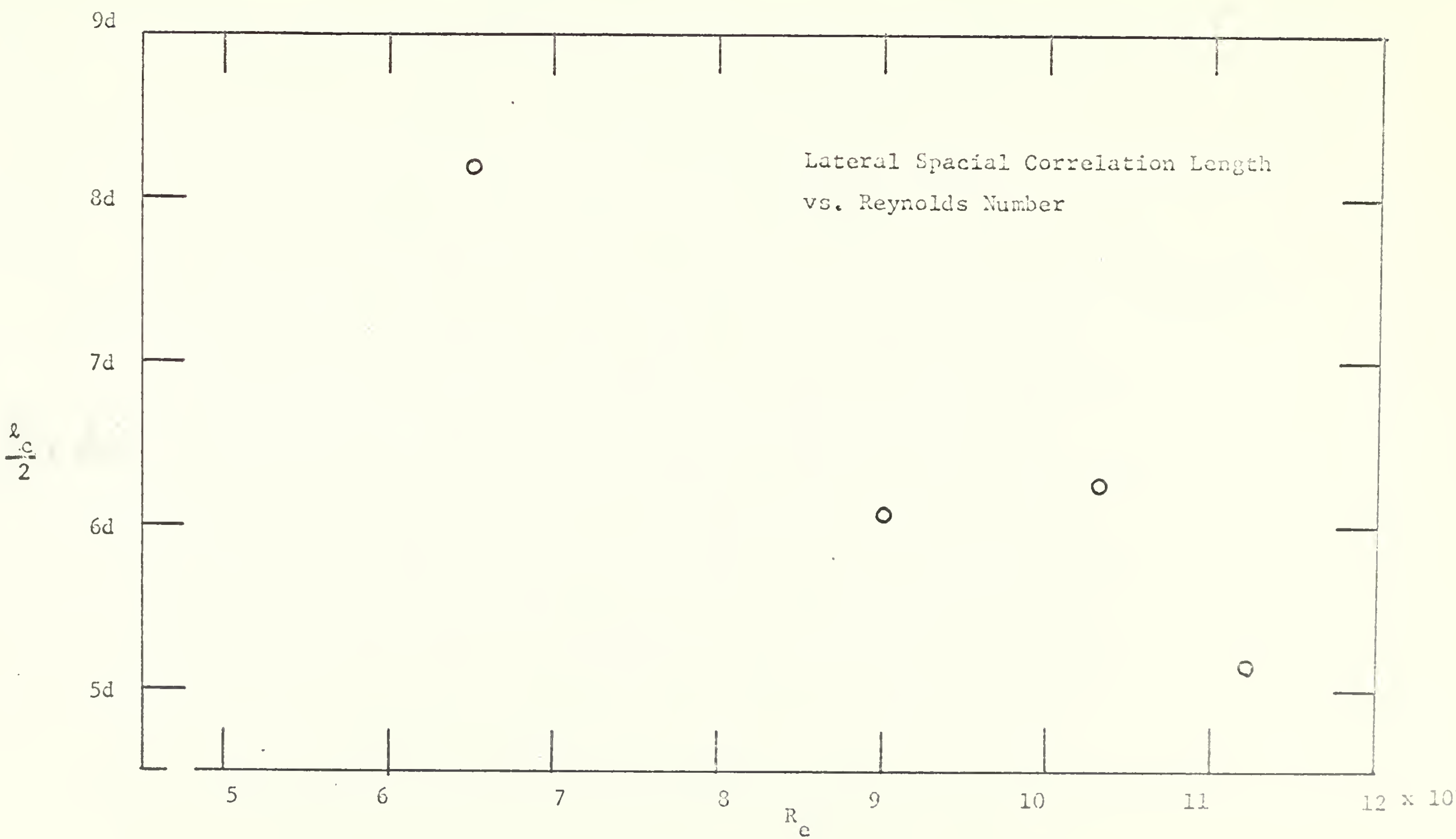

Lateral Spacial Correlation Length
vs. Reynolds Number
9d
8d
7d
6d
5d
$\frac{\ell_c}{2}$
5
6
7
8
9
10
11
12 x 10^3
R_e

FIGURE 3

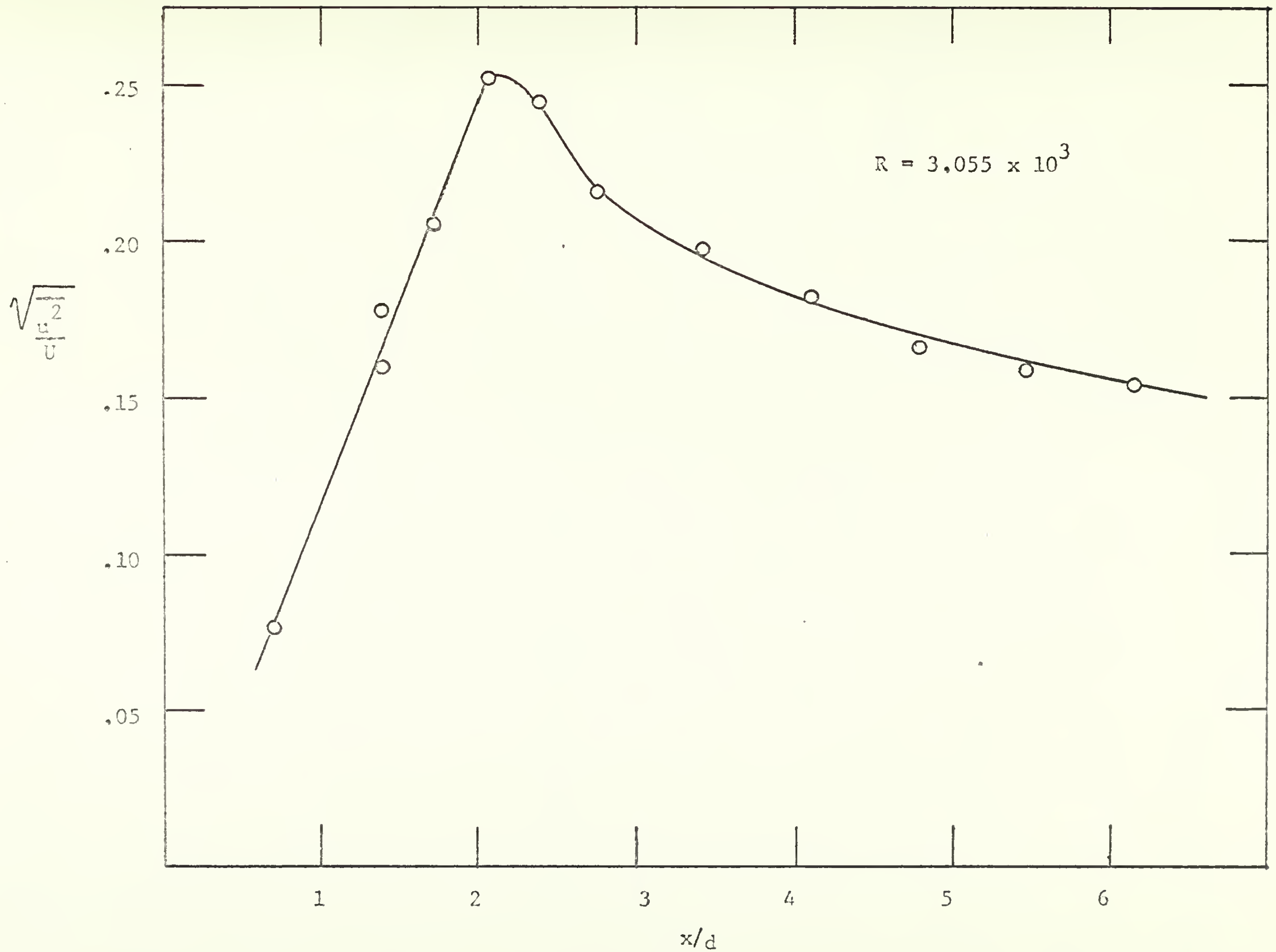


FIGURE 9

MAXIMUM RMS VELOCITY AS A FUNCTION OF DISTANCE DOWNSTREAM

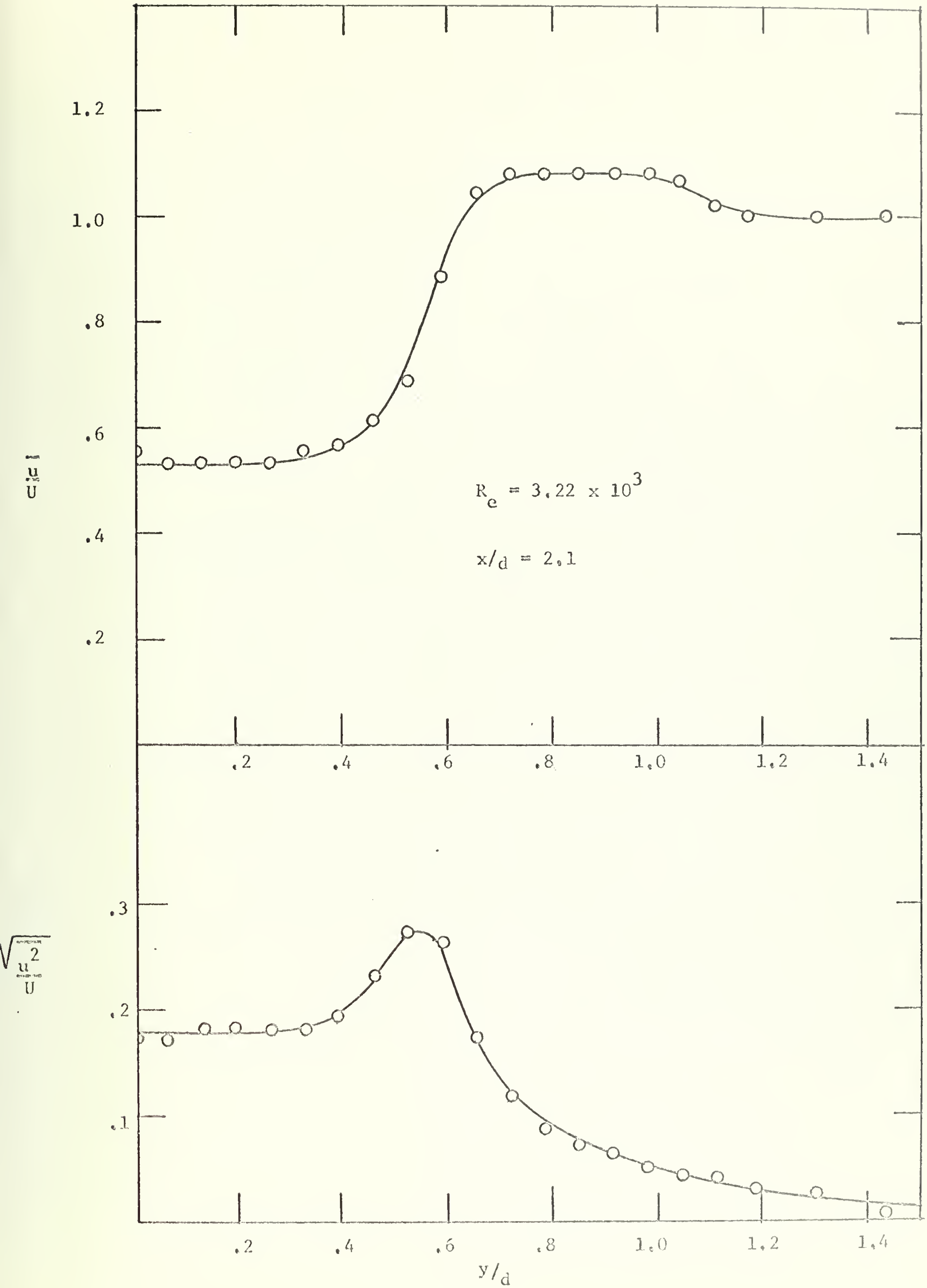


FIGURE 10

RMS VELOCITY AND MEAN VELOCITY AS A FUNCTION OF y

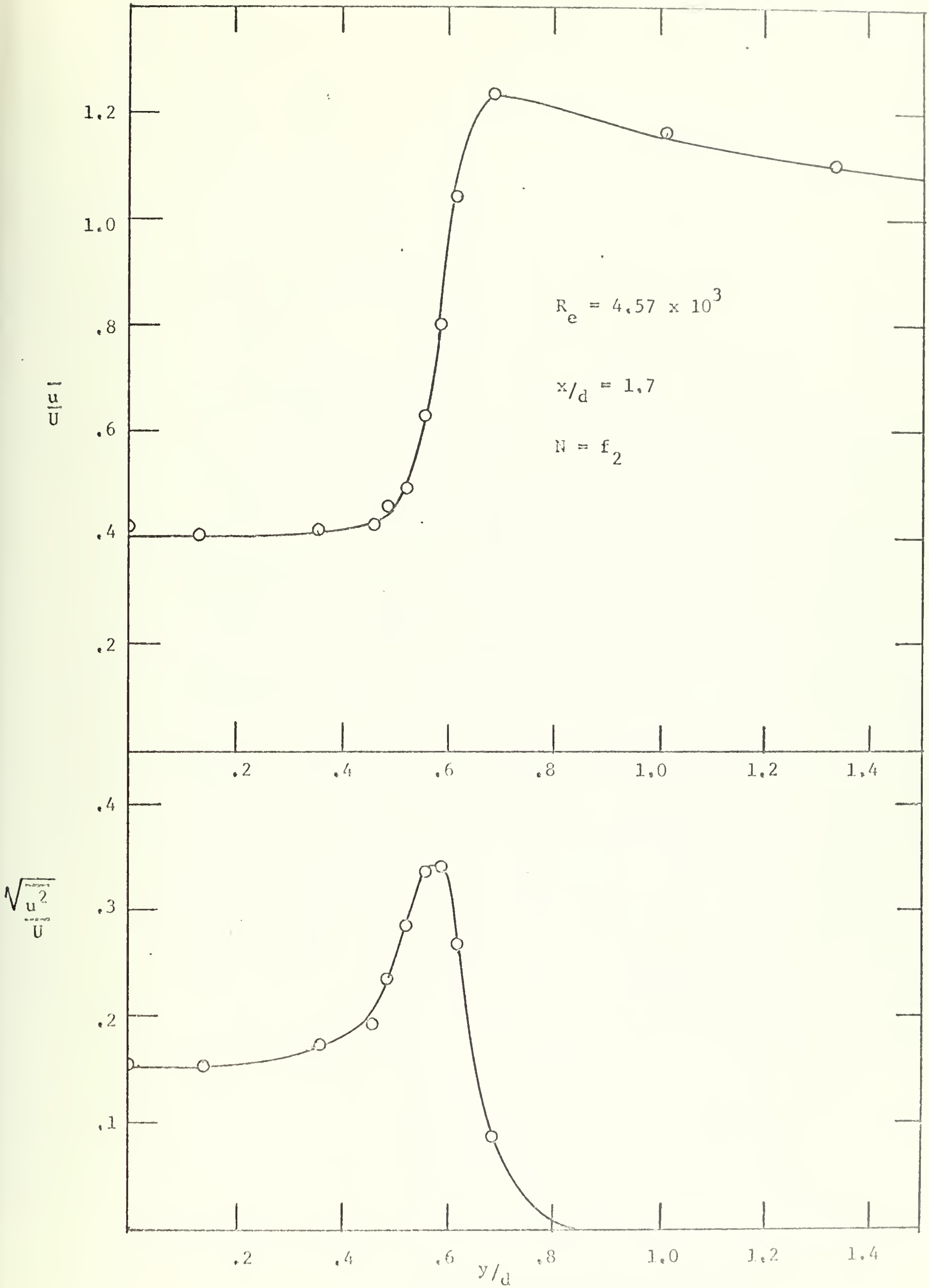


FIGURE 11

RMS VELOCITY AND MEAN VELOCITY AS A FUNCTION OF y

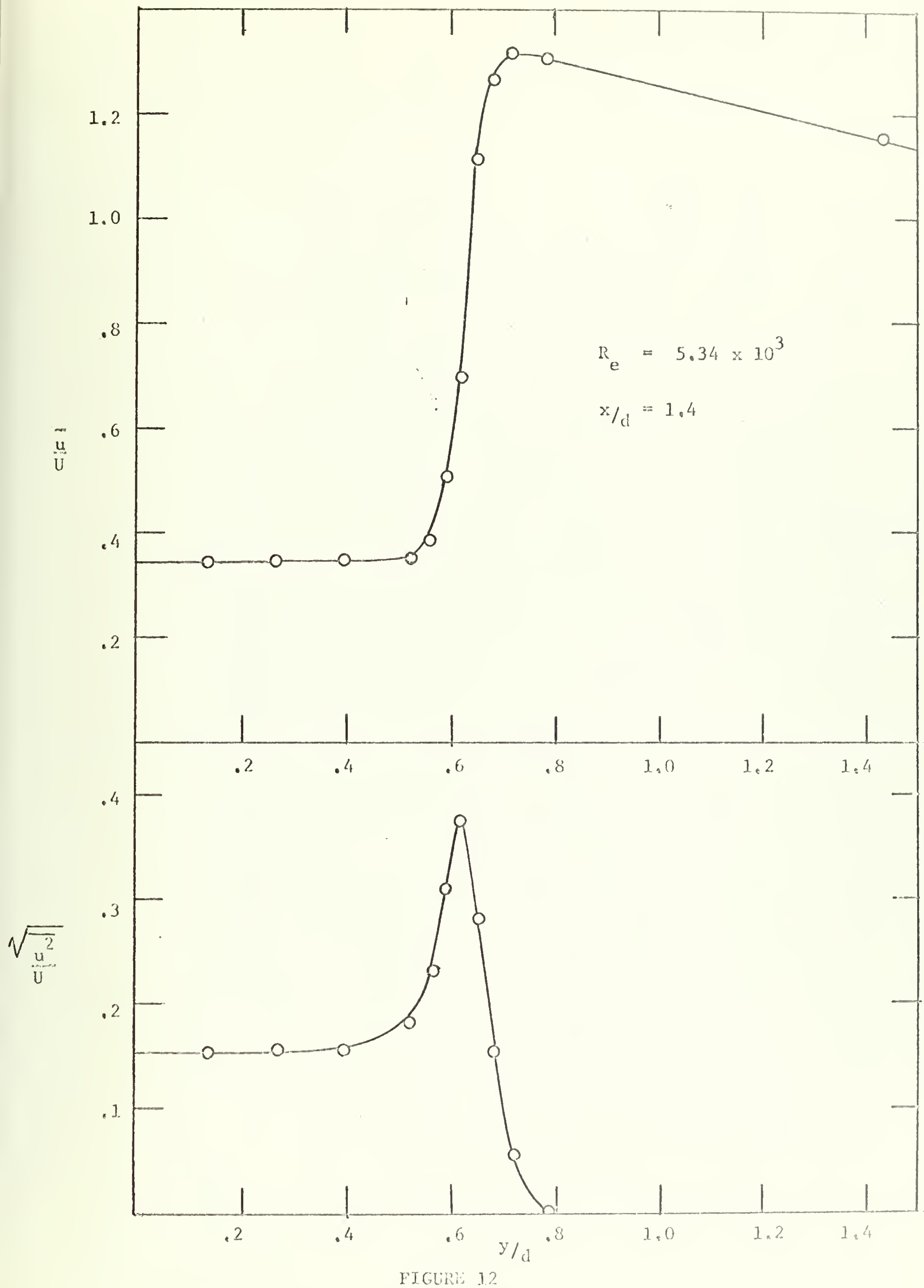


FIGURE 12

RMS VELOCITY AND MEAN VELOCITY AS A FUNCTION OF y

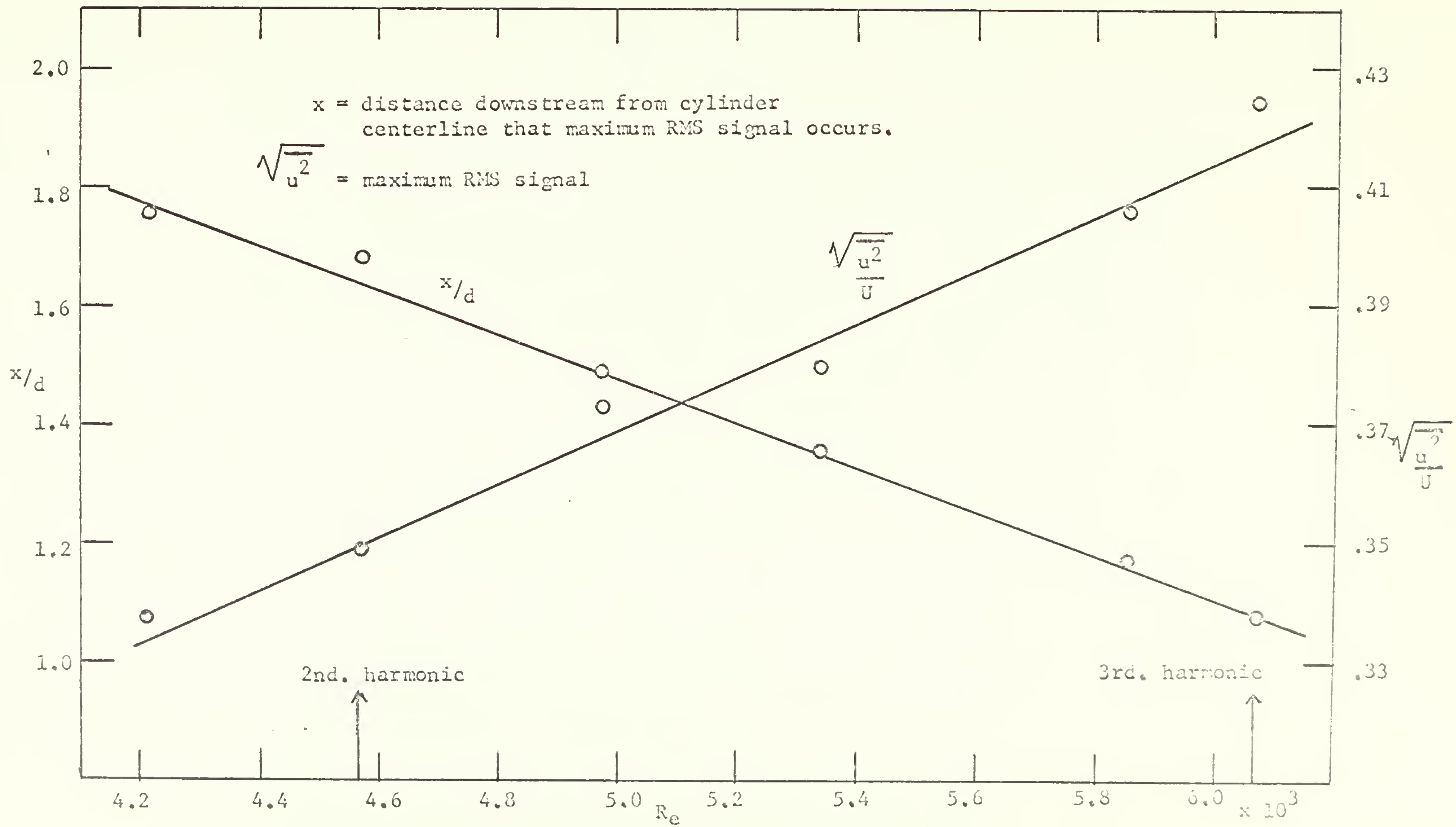


FIGURE 13

DISTANCE TO "ROLLING UP" OF VORTEX AND MAXIMUM RMS VOLTAGE AS A FUNCTION OF REYNOLDS NUMBER

APPENDIX B

EQUIPMENT USED FOR CORRELATION LENGTH MEASUREMENTS

Thermo-Systems, Inc.

Model 1010A Constant Temperature Anemometer

Model 1015B Correlator

Model 1031-2 Monitor and Power Supply with

(2) Model 1035 Linearizer modules.

The sensing device used was a Tungsten Hot Wire .00015" diameter and .05" long.

The output of the Constant Temperature Anemometer was led into the linearizer modules so that given fluctuations in velocity would give the same voltage fluctuations for any mean velocity. The linearized output was sent to the correlator unit that summed and differenced the two signals; also the separate signals were obtained from the output of this unit. The output from the correlator was sent to the Brüel and Kjaer Frequency Analyser connected to the Graphic Level Recorder on which the signals were recorded in db level. See Figure [3]. The output was continuously observed on an oscilloscope. This was converted to RMS voltage and used in the cross-correlation coefficient equation. The magnitude of fluctuating velocity was not computed as the conversion factor cancels out of the R(z) equation.

The free stream velocity of the wind tunnel was obtained by placing the hot wire sensing device in a free stream portion of the open test section and recording the output voltage from the linearizer to use on the previously obtained calibration curve. From this velocity, together with the cylinder diameter and kinematic velocity, the Reynolds number was computed.

The method used to check for coincidence of frequencies was to run a frequency analysis of the output of an accelerometer mounted on a support of the wire. When the two frequencies coincided, the natural frequency would be reenforced to its highest value. A more accurate method of matching frequencies was devised described on page (8).

APPENDIX C

RECOMMENDATIONS

1. Take correlation coefficient measurements at different locations along the cylinder to determine if there is spacial homogeniety.
2. Work with a system that will allow a greater amplitude of cylinder vibration to determine how amplitude of vibration affects the correlation length. The correlation lengths should increase due to a coupling of mechanical and hydrodynamic forces. This could be accomplished by increasing the diameter of the cylinder without increasing the mass very much. This also would allow a coincidence of frequency at the 1st. harmonic.
3. Take more correlation measurement in close proximity to the point of coincidence of frequencies to determine if there is any deviation from the ℓ_c vs. Reynolds number trend observed in other regions of frequency.
4. Take correlation measurement data in the same Reynolds number range that el Baroudi did to see what changes in trends or values are observed for a non-rigid cylinder.

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